International Baccalaureate: Physics Extended Essay

Calculation and evaluation of the age of the universe using Hubble's law

Abstract

The following essay attempts to estimate and evaluate the age of the universe using Hubble's law and images from the Hubble space telescope.
The calculation of the age of the universe was attained through the specific calculation of the distance, angular size, redshifts and hence the escape velocity of ten galaxies. These calculations were used to attain Hubble's parameter; a ratio between the escape velocity of a galaxy and its distance with respect to earth which was averaged from the ten separate evaluations. An evaluation of the age of the universe was attained through the reciprocal of Hubble's parameter. The value that was produced in this essay is a rough estimate of the age of the universe and dates the cosmos to be approximately $13.086 \times 10^9 \pm 3.0604 \times 10^9$ years old. This is a rough value not only because of the rather large uncertainty of this specific calculation but rather because it does not take into account a crucial factor that is commonly accepted among the science community for the past decade, the universe is expanding at an accelerating rate. Practically, if the expansion of the universe is increasing the recessional velocity of a distant galaxy would be relatively smaller than the velocity predicted by Hubble's law and vice versa (in other words a galaxy with a given recessional velocity will be farther away than expected in the given case of an accelerated expansion). Hence, an accelerating universe is in fact older than a universe that is expanding at a constant rate therefor although this investigation does not produce an accurate estiamtion of the age of the universe it manages to provide a feasbale lower limit which is significant in its own right.

Word count: 285

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1. Introduction

The following essay attempts to estimate and evaluate the age of the universe using Hubble's law and images from the Hubble space telescope. The scope of this research goes beyond attaining a value for the age of the universe as it examines the validity of such a value attained by the now outdated Hubble's law through comparison to newer theories and recently collected data.

2. Background Information and Literature

The "Big Bang" theory is a cosmological theory regarding the origin of the universe; it claims that the universe originated from a single point in somewhat of a huge "explosion" and it has been constantly expanding ever since.

Up to the twentieth century it was commonly accepted that the universe's existence is eternal. However, in the late 1920's Edwin P. Hubble (1953-1889), an American astronomer considered a pioneer of modern cosmology, discovered the expanding nature of galaxies away from one another and thus discovered that the universe was in fact expanding. From his position at the Mount Wilson Observatory Hubble observed distant galaxies and in 1926 published a classification method for galaxies[1]. He then shifted his attention and research towards a known discovery that was lacking an explanation. The American astronomer Vesto Slipher (1875-1969) declared that the spectrum of certain nebulas shows the odd phenomenon known as a "red shift"[2]. Hubble implemented spectroscopy methods for galactic examinations and in 1929 stated that galaxies are moving apart at speeds (v) that increase in direct proportion to their distance (d) from the observer - $\boldsymbol{v} \propto \boldsymbol{d}$. This was a first of a kind observation that later served as a basis for the "Big Bang" theory: if all galaxies are moving apart from one another then they all originated from a singular point[3]. Hubble's findings where the first to reinforce the inevitable conclusion of Albert Einstein's (1879-1955) general theory of relativity which clearly implies the existence of a dynamic universe.

Hubble's parameter (H_0) represents the ratio between the speed at which the galaxy is moving and its distance from the observer. Because it is a ratio and not a constant as it is commonly believed to be [it does not only change as time goes by (therefore it is referred to as H_0) it also changes when vast distances are involved], there is no absolute value of the parameter however its value is estimated to range between 50 to 90 kilometers per second/mega parsec; 72 ± 8 km/sec/Mpc (2000)[4] is the latest and most broadly accepted value of our day. This outstanding discovery provided modern cosmologists an opportunity to evaluate the age of the universe using a mathematical model. There are several mathematical models that are capable of evaluating the age of the universe each taking in account unique limiting circumstances. The following research will not only revolve around the calculation of a current value of Hubble's parameter and hence an evaluation of the age of the universe, but will also attempt to study and evaluate the efficiency of using Hubble's law and theory in order to determine the age of the universe.

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[1] Annex 3 - Hubble Galaxy Classification Scheme (pg. 15).
[2] Doppler Effect (pg. 4).
[3] Annex 1 - Cosmic expansion process following the "Big Bang" (NASA) (pg. 13).
[4] "Hubble Space Telescope Key Project to Measure the Hubble Constant" (Dec. 2000) - Wendy L. Freedman.

3. Apparatus

Equipment:

- 18 × same scale galaxy images (IASA)[5].
- 18 × H-a spectra.
- 18 × Ca-H spectra.
- 18 × Ca-K spectra.
- 1 × image processing software (UW)[6].
- 1 × Photoshop[7] software.

4. Method

In order to follow the theoretical model above one must first determine Hubble's parameter. The following presents this procedure:

1. *Galaxy Selection*

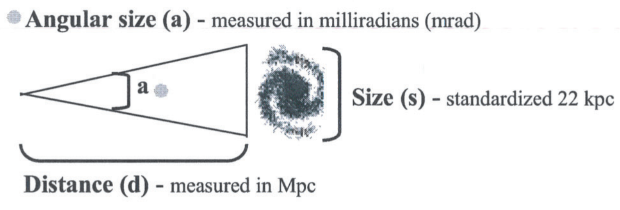
In order to formulate a common base and thus minimize the possible range of errors throughout the data collection process all eighteen galaxies (all measurements in the body of the essay refer only to the ten "best fit" galaxies that were selected after all data was processed and produced a value of Hubble's parameter close to the reference value of 72 ± 8 km/sec/Mpc) used in this essay are type Sa spiral galaxies by definition of the Hubble Classification Scheme[8].

2. *Distance Calculation*

According to the Hubble Classification Scheme all galaxies of similar type have a similar size; the average size of a type Sa spiral galaxy is 22 kpc. The measuring of the galaxy's distance (with respect to earth) will be calculated (all distance related values are given to one significant figure) according to its angular size by the following formula (applicable for small angles):

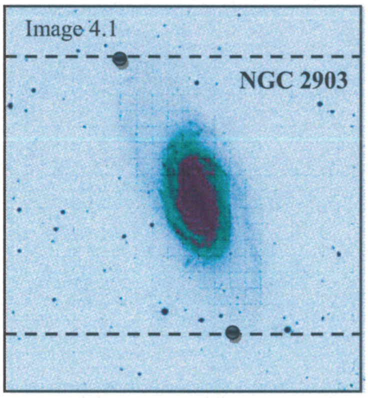
$$a = \frac{s}{d}$$

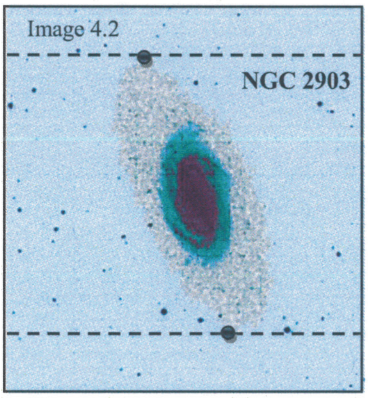
$$\therefore d = \frac{s}{a}$$



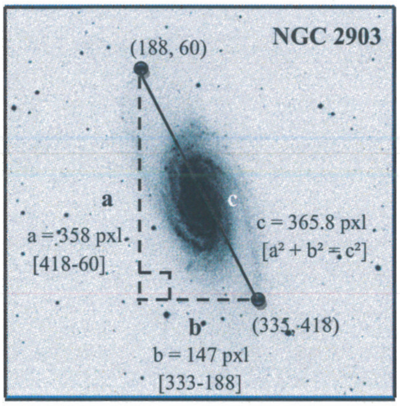
[5] All galaxy images and corresponding spectrums used in this essay are courtesy of IASA - Israel Arts and Science Academy.
[6] Image processing software courtesy of the University of Washington's astronomy department.
[7] Photoshop
[8] Annex 3 - Hubble Classification Scheme (pg.13).

An interior measurement included in the distance calculation process is the calculation of the angular size of the galaxy which is necessary for the above formula.
The galaxy images used in this essay were taken in black and white. Via the usage of "Photoshop" the gray levels are translated into color. A tone selected from the given color scheme serves as a color barrier that indicates the outline of a galaxy. The following is a demonstration of this mapping method:





Red 188, green 227 and blue 225 is the unified tone combination which indicates a galaxy border in this investigation (images 4.1, 4.2).
After determining the boundaries of the galaxy its angular size is determined using Pythagoras's theorem and the datum-points [in pixels (pxl)], that are calculated by the image processing software, in the following manner:



This procedure is only partial and in order to convert the galaxy's size in pixels into radians and thus determine the scale of the telescope used and its detector we must calculate the radial value of a single pixel in this specific image. This point of reference is created by comparing the previously established size of a galaxy with its pixel size. NGC 2903 apparent dimension is 12.6 x 6.6 (arc min), 12.6 arc min along the long axis (in the image represented by c). Multiplying this value by the number of radians per arc min (approximately 0.00029) and dividing the product by the galaxy's measured size in pixels will yield the scale of the given telescope:

$$\frac{12.6\ arc\ min\ \times 0.00029}{395\ pxl} = 9.3\ \times 10^{-6}\ rad\ per\ pxl$$

It is important to note that although a system was implemented in order to standardize the measuring of the angular size and make it as least arbitrary as possible, and thus maximize the accuracy of the calculations, there is still an element of uncertainty to be taken in account. This uncertainty is derived from the manner in which this data was collected (manually) and it has a magnitude of ± 2 pixels. Hence, the angular size has an uncertainty of $\pm 1.9 \times 10^{-5}$ radians. However, there is no uncertainty to take into consideration in the overall distance value for this uncertainty is insignificant in its magnitude.

Name	**Angulr Size** a / mrad $\Delta a \pm 1.9 \times 10^{-5}$ rad	**Distance** d / Mpc
NGC 1357	0.80	27.500
NGC 2276	0.77	28.571
NGC 3147	0.96	22.910
NGC 3368	1.87	11.765
NGC 3627	2.40	9.167
NGC 6181	0.72	30.550
NGC 6643	0.98	22.449
NGC 1832	0.79	27.848
NGC 2775	1.19	18.487
NGC 5248	1.37	16.058
NGC 6764	0.71	30.986

3. *Spectrum Analysis and Recessional Velocity Calculation*

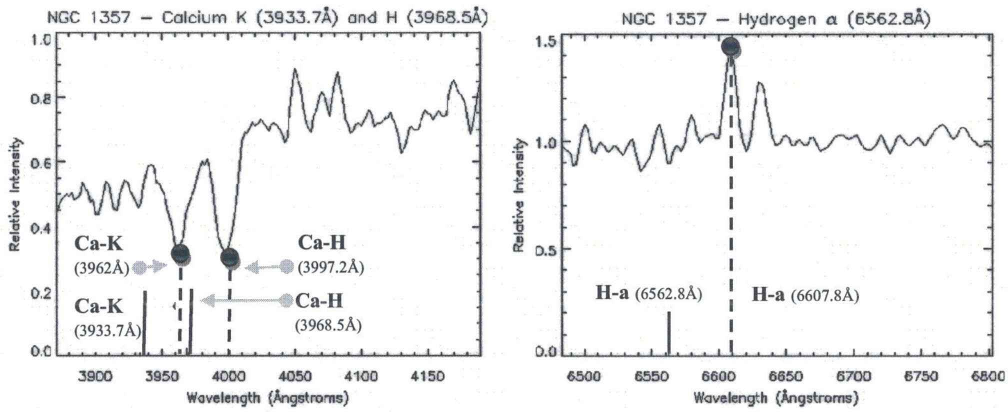
The recession of the galaxies from the earth (serves as a stationary observer) causes a shift in their observed spectra, this effect is known as the Doppler Effect[9], with spectral features (e.g. absorption and emission lines) being shifted "redwards" (towards longer wavelengths) from the rest frame wavelengths [measured in Angstroms (Å)]. The amount of this shift, and hence the recession velocity, is measured in terms of the redshift parameter (z). The aforementioned procedure was repeated three times per galaxy each measurement based on three different spectral lines [Ca-K (3933.7 Å), Ca-H (3968.5 Å), H-a (6562.8 Å)].

[9] Christian Andreas Doppler (1803-1853)

Exemplary analysis:

-- Measured wavelength

— Rest (original) wavelength



Using the spectrum analysis we determine the galaxies shifted wavelength (per spectrum) and insert it into the basic Doppler formula which determines the speed of the object via the use of its redshift and the speed of light (c)[10]:

1. $z = \frac{\lambda - \lambda_0}{\lambda_0}$

2. $z = \frac{v}{c}$

$\therefore v = z \times c$

λ : Measured wavelength

λ_0: Rest (original) wavelength

v : Recessional velocity

So as to produce a single recessional velocity the average of the three redshift values, per galaxy, was calculated. Each average has an individual uncertainty that was determined by dividing the range of the redshift values by two (the uncertainties of the recessional velocity were determined in the same manner):

$$\Delta z = \pm \frac{z_{max} - z_{min}}{2}$$

[10] The absolute value of the speed of light used in this essay is 3×10^8 m/sec.

Name	Measured Wavelength Å			Redshift				Velocity km/sec
	Ca-K	Ca-H	H-a	Ca-K	Ca-H	H-a	Ave.	
NGC 1357	3962.0	3997.2	6607.8	0.0071942446	0.0072319516	0.0068568294	0.0070943419 $\Delta z \pm 5.904 \times 10^{-4}$	2128.303 $\Delta v \pm 269.384$
NGC 2276	3966.0	3996.4	6614.9	0.0082110990	0.0070303641	0.0079386847	0.0077267159 $\Delta z \pm 1.876 \times 10^{-4}$	2318.014 $\Delta v \pm 56.280$
NGC 3368	3944.9	3980.0	6583.6	0.0028471922	0.0028978203	0.0031693789	0.0029714638 $\Delta z \pm 1.611 \times 10^{-4}$	891.439 $\Delta v \pm 48.330$
NGC 3627	3942.5	3977.7	6578.2	0.0022370796	0.0023182563	0.0023465594	0.0023006318 $\Delta z \pm 5.474 \times 10^{-5}$	690.189 $\Delta v \pm 16.422$
NGC 6181	3965.2	4000.3	6610.2	0.0080076281	0.0080131032	0.0071920522	0.0077376278 $\Delta z \pm 4.105 \times 10^{-4}$	2321.288 $\Delta v \pm 123.150$
NGC 6643	3951.9	3987.0	6593.8	0.0046266873	0.004661711	0.0047235936	0.004670664 $\Delta z \pm 4.845 \times 10^{-5}$	1401.199 $\Delta v \pm 13.995$
NGC 1832	3961.3	3995.6	6607.0	0.0070262951	0.006827766	0.0067349302	0.0068600006 $\Delta z \pm 1.407 \times 10^{-4}$	2058.000 $\Delta v \pm 42.210$
NGC 2775	3954.2	3988.6	6593.0	0.0052113786	0.005064886	0.0046016944	0.0049593197 $\Delta z \pm 3.048 \times 10^{-4}$	1487.796 $\Delta v \pm 91.440$
NGC 5248	3945.7	3981.6	6586.0	0.0030505631	0.0033768459	0.0035350765	0.0033208285 $\Delta z \pm 2.423 \times 10^{-4}$	996.248 $\Delta v \pm 72.690$
NGC 6764	3960.5	3995.6	6610.2	0.0068129242	0.0068287766	0.007222527	0.0069557426 $\Delta z \pm 2.033 \times 10^{-4}$	2086.723 $\Delta v \pm 60.990$

4. *Calculation of Hubble's Parameter*

As was previously stated the equation that represents Hubble's law is the following:

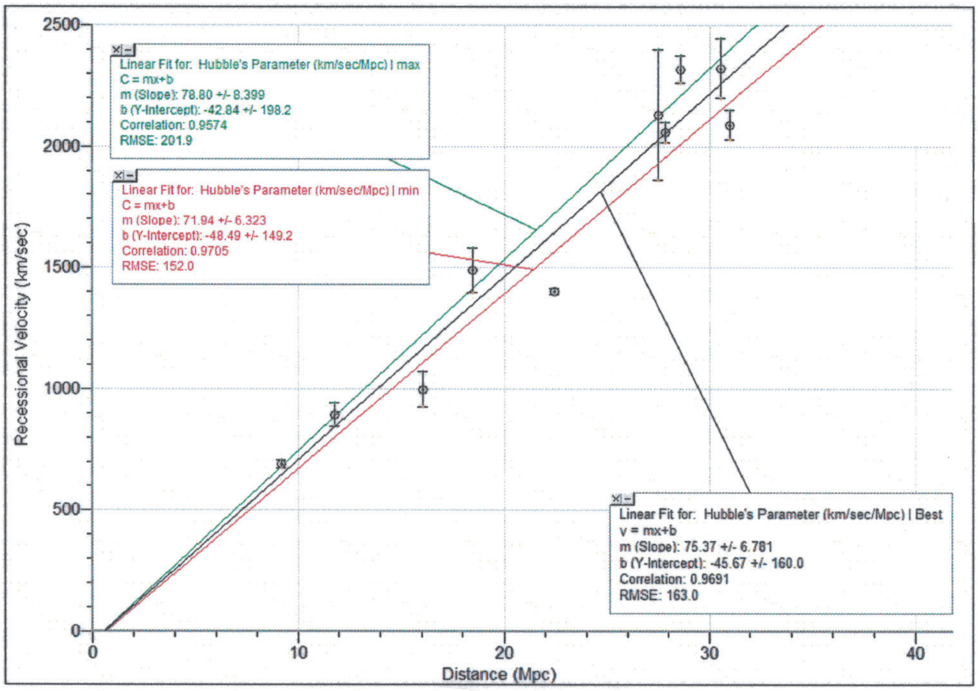
$$v = H_0 \times d$$
$$\therefore H_0 = \frac{v}{d}$$

Each measurement of Hubble's Parameter has an individual uncertainty that was determined in the same manner as the redshift uncertainties. The absolute value that was achieved is the average of the ten final measurements and the value of uncertainty is the full range of measurements [maximum including maximum value of the highest measurement (measurement + individual uncertainty) and vice versa] divided by two.

Name	Distance d / Mpc	Velocity km/sec	Hubble's Parameter H_0 / km/sec/Mpc	
			Measurements	Ave.
NGC 1357	27.500	2128.303 $\Delta v \pm 269.384$	77.393 $\Delta H_0 \pm 9.795$	73.174 $\Delta H_0 \pm 8.266$
NGC 2276	28.571	2318.014 $\Delta v \pm 56.280$	81.132 $\Delta H_0 \pm 1.969$	
NGC 3368	11.765	891.439 $\Delta v \pm 48.330$	75.770 $\Delta H_0 \pm 4.108$	
NGC 3627	9.167	690.189 $\Delta v \pm 16.422$	75.291 $\Delta H_0 \pm 1.791$	
NGC 6181	30.550	2321.288 $\Delta v \pm 123.150$	75.971 $\Delta H_0 \pm 4.031$	
NGC 6643	22.449	1401.199 $\Delta v \pm 13.905$	62.417 $\Delta H_0 \pm 0.553$	
NGC 1832	27.848	2058.000 $\Delta v \pm 42.210$	73.901 $\Delta H_0 \pm 1.516$	
NGC 2775	18.487	1487.796 $\Delta v \pm 91.440$	80.477 $\Delta H_0 \pm 4.946$	
NGC 5248	16.058	996.248 $\Delta v \pm 72.690$	62.041 $\Delta H_0 \pm 4.527$	
NGC 6764	30.986	2086.723 $\Delta v \pm 60.990$	67.344 $\Delta H_0 \pm 1.968$	

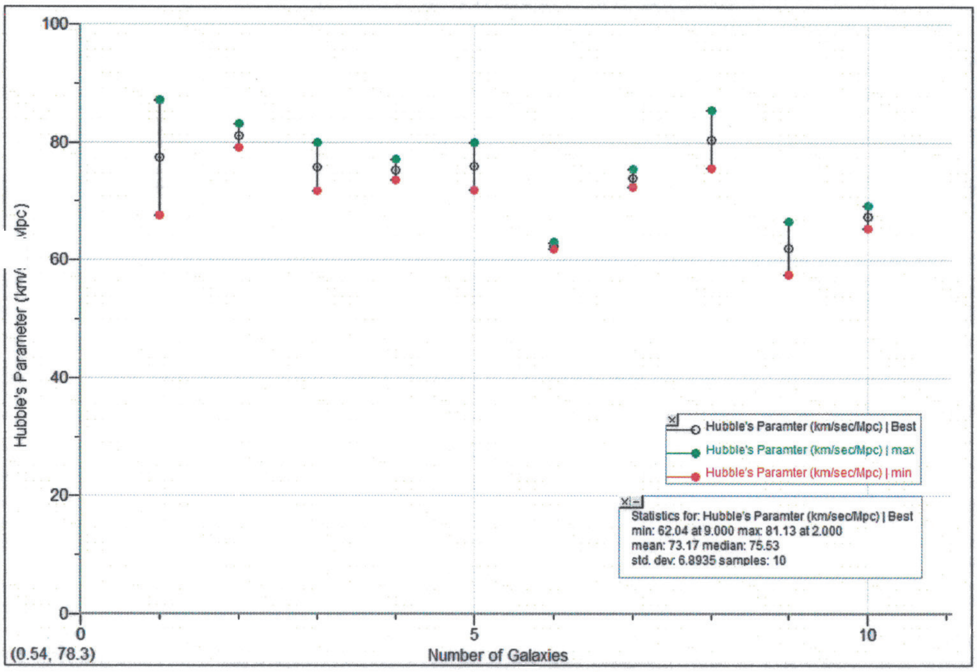
The following graphs where produced through Vernier's "Logger Pro 3.4.6" software. The graph below demonstrates the linear relationship (porportional relationship) between the galaxy's distance from the earth and its recessional velocity. Using the "Linear Fit" application a trendline was produced for this set of data; this relationship is in fact the value of Hubble's parameter. Additional trendlines were produced for the maximum values of the individual measurments which are representative of the maximum possible trendline and vice versa.

Graph 1 - Recessional Velocity (km/sec) against Distance (Mpc)



It is evident from the graph above that a fitting relationship has been found, according to Hubble's law a linear relationship is to be produced from the quotient of the recessional velocity against distance and the linear relationship that has been obtained in this research has a correlation of 0.9691 quite close to the maximum correlation of 1.000. Yet the graph also shows evidence of a distinct systematic error. According to theory the trendline of the graph is to pass through the origin, for at zero distance there is to be an equivalent recessional velocity equal to zero [the best trendline intercepts with the y axis at approximately (0, -45.67)]. Despite this seemingly drastic systematic error when comparing the mean average of Hubble's Parameter (graph 2) produced in this essay (73.174 ± 8.266 km/sec/Mpc) with what is considered the most accurate evaluation of the parameters value today (72 ± 8 km/sec/Mpc) there is only a minor difference of 1.174 km/sec/Mpc in addition to a relatively small uncertainty.

Graph 2 – Hubble's Parameter (km/sec/Mpc)



The previously deployed procedures where merely necessary measurements in the course of calculating the age of the universe. The following theoretical experiment demonstrates the logic behind the basic mathematical model for the age of the universe. A certain galaxy is at a distance d from the observer, its velocity is thus $v = H_0 \times d$. Assuming that the galaxy and the earth originated from the same singular point and were at time zero at zero separation from one another than by basic kinematics the time taken (t) to achieve the current separation (d) is given by the following formula:

$$v = \frac{d}{t}$$

$$v = H_0 \times d$$

$$\therefore t = \frac{1}{H_0}$$

In order to achieve a time estimate for the age of the universe in years it is necessary to convert the units in the Hubble equation from seconds to years in addition to cancelling distance units.

Hubble's Parameter's inverse:

$$\frac{1}{73.174 \times kmsec^{-1}Mpc^{-1}} =$$

$$= \frac{1}{73.174 \times 10^3\ msec^{-1}} \times 10^6\ pc$$

$$= \frac{1}{73.174 \times 10^3\ msec^{-1}} \times 10^6 \times 3.086 \times 10^{16}\ m$$

$$= 4.217 \times 10^{17}\ sec$$

$$= \frac{4.127 \times 10^{17}\ sec}{31536000\ \text{secyr}^{-1}}$$

$$= 13.086 \times 10^9 \pm 3{,}060{,}403{,}700\ y$$

1 pc = 3.086 m

1 yr = 31536000 sec

5. Conclusion

The value that was previously produced is a rough estimate of the age of the universe and dates the cosmos to be approximately $13.086 \times 10^9 \pm 3.0604 \times 10^9$ years old. This is a rough value not only because of the rather large uncertainty of this specific calculation but rather because it does not take into account a crucial factor that is commonly accepted among the science community for the past decade, the universe is expanding at an accelerating rate. From recent observations it has been shown that the ratio known as Hubble's Parameter is in fact constantly increasing as opposed to the conjecture derived from Einstein's general theory of relativity which predicts that due to the gravitational pull between galaxies it should theoretically decrease. Only in 1998 did a team of researchers[11] discover evidence for the repulsive side of gravity (that is in fact noted in Einstein's general theory of relativity which accepts the possibility of forms of energy with strange properties producing repulsive gravity, this however was regarded as purely theoretical until the 1998 discovery). Via the careful observation of distant Type Ia supernovae astronomers found that they were fainter than expected. The most plausible explanation for the discrepancy is that the light emitted from the supernovae traveled a greater distance than theorists had predicted. The current popular model for cosmic expansion implies that the universe has not always been accelerating. In brief, if the universe had been constantly accelerating modern science could not account for the existence of cosmic structures observed in the universe. According to cosmological theory cosmological structures such as galaxies evolved from small inhomogeneities in the matter density of the early universe, this strong attractive gravity of over dense regions stopped their expansion thus allowing them to form (evidence for this process can be found in variations in the temperature of CMB[12]). Practically, if the expansion of the universe is increasing the recessional velocity of a distant galaxy would be relatively smaller than the velocity predicted by Hubble's law and vice versa (in other words a galaxy with a given recessional velocity will be farther away than expected in the given case of an accelerated expansion). Hence, an accelerating universe is in fact older than a universe that is expanding at a constant rate[13] therfore the previously achieved value can only serve as a lower limit estimate for the age of the universe.

Prior to the 1998 discovery there were two known factors with direct influence on the nature of the universe's expansion - vacuum energy density derived from Einstein's "cosmological constant" (ρ_Λ) and mass related energy density (ρ_m). Classic cosmology produced three possible scenarios of the nature of the universe's expansion, each predicting a different age of the universe (insert models or refer to annex?). However, in order to produce the observed acceleration of the universe there is a need of an additional energy source that will balance the attractive side of gravity (i.e. a repulsive side of gravity), commonly referred to as dark energy. There are several theoretical models each predicting a different transition point between a decreasing expansion of the universe to an accelerating expansion, however these will not be explored in this investigation.

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[11] A. Riess, A. Filippenko et al. (High-Z Supernova Search), Astron. J. 116, 1009 (1998) S. Perlmutter, G. Aldering, G. Goldhaber et al. (Supernova Cosmology Project), Astrophys. J. 517, 565, (1999).

[12] CMB - Cosmic Microwave Background.

[13] Annex 2 - Model of cosmic expansion (pg. 14).

6. Evaluation and Points of Improvement

Although the majority of the measurements in this investigation have a relatively high precision the final uncertainty which is a mathematical summation of all minor uncertainties is rather high and has a magnitude of $\pm 3.0604 \times 10^9$ years which is equivalent to an uncertainty of $\pm 23.387\% \approx \pm 23\%$, a considerably large and significant uncertainty. However, a higher level of accuracy could not be attained under the circumstances of the experiment for all measurements were taken manually and thus have and additional factor of human error. It is important to note that certain measures were taken in order to minimize the uncertainties in the spectrum measurements such as repeated measurements and an elimination process that was previously deployed in section 4.1. In addition, a computer generated mapping system was created specifically for this investigation thus significantly decreasing the uncertainties in the angular size calculations and hence in the distance calculations as a whole.

A major question arises throughout the evaluation process of this investigation and that is of the relevance of such research in a time in which Hubble's law has been proven wrong. While this investigation does not produce an accurate estiamtion of the age of the universe it manages to provide a feasbale lower limit which is significant in its own right.

Bibliography

Hogan, Craig J., Kirshner, Robert P. and Suntzeff, Nicholas B. "Surveying Space-time with Supernovae" Scientific American. Jan. 1999.

Riess, Adam G., Turner Michael S. "From Slowdown to Speedup" Scientific American. Feb. 2004

Perlmutter, Saul. "Supernovae, Dark Energy, and the Accelerating Universe" Physics Today. Apr. 2003.
http://www.physicstoday.org

Gorelik , Ivan ." The Hubble Constant as the Angular Velocity of Light in the Closed Universe" Space Genetics. 17 Nov. 2008.
http://www.geocities.com/Area51/Nebula/3735/dates.html

Darby, Dyar. "How do we know that the universe is expanding?" Mount Holyoke - Planet Earth, Astronomy/Geology 104, spring 2006, Departments of Astronomy and Earth and Environment. 31 Jan. 2006.
http://www.mtholyoke.edu/courses/mdyar/geo104/hw1_redshift/redshift_hw.html

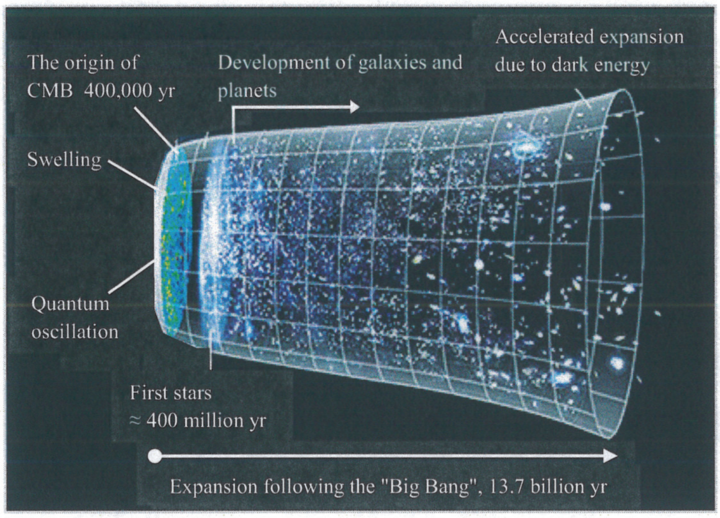
Plionis, M. "The Quest for the Cosmological Parameters" Institute of Astronomy & Astrophysics, National Observatory of Athens, 15236, Athens, Greece. Nov. 2002.
http://arxiv.org/PS_cache/astro-ph/pdf/0205/0205166v2.pdf

Zeilik, Michael. Astronomy the Evolving Universe Ninth Edition. Cambridge University Press, 2002.
http://books.google.co.il/books?id=HQJ0rtsy8u0C&pg=PA419&lpg=PA419&dq=NGC+1357+distance+and+speed&source=bl&ots=afEIacNVp&sig=zQbU9MZllq,nfowM9YeVJosk6mu4&hl=iw&ei=odZRStm3N4Oi_AbKzMHEBQ&sa=X&oi=book_result&ct=result&resnum=1

"Dark Matter" The Encyclopedia for Astronomy, Astrophysics and Space Science; Tel Aviv University Astronomy Club- Tau AstroClub. 1 Oct. 2009.
http://astroclub.tau.ac.il/astropedia/Hubble_Parameter.html

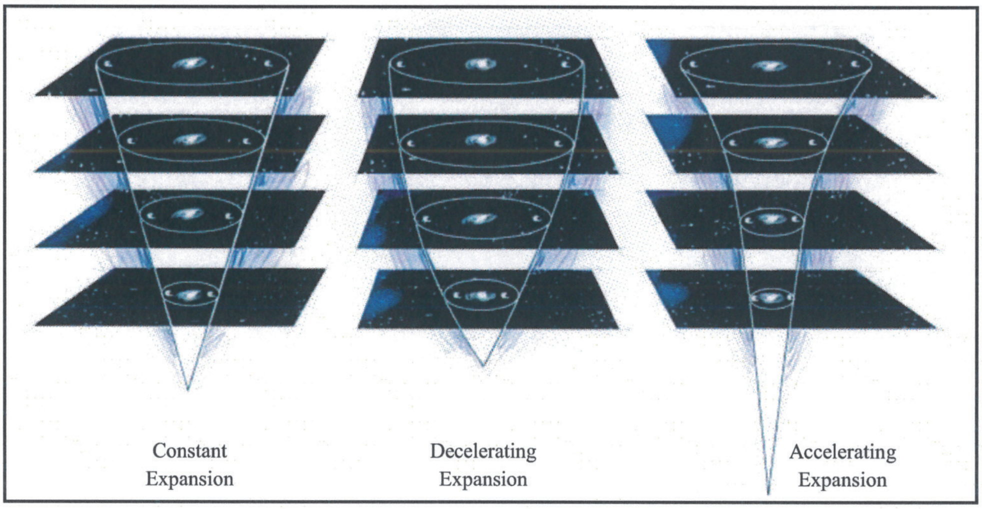
"Determining the Hubble Constant" University of Washington - "Exploring the Dark Universe". 1999.
http://www.astro.indiana.edu/darkuniverse/Hubbleconstantc.doc

Annex 1 - *The universe's expansion process following the "Big Bang" (courtesy of NASA).*



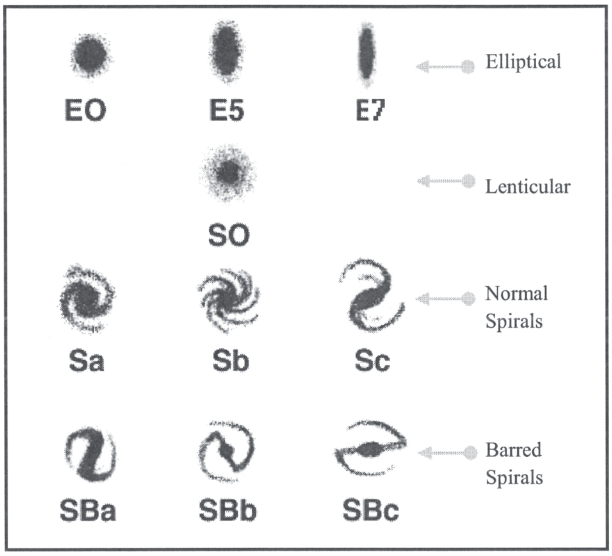
Annex 2[14] - *Models of Cosmic Expansion*

Cosmic expansion could, in theory, follow one of three patterns: it may be constant (left), decelerating (center) or accelerating (right). In each case, a given portion of the universe grows in size as time passes (from bottom to top). The illustration demonstrates the inevitable conclusion that an accelarating universe is in fact older than a a universe expanding at a constant or decelerating rate (see section 5).



[14] Riess, Adam G., Turner Michael S. "From Slowdown to Speedup" Scientific American. Feb. 2004

Annex 3 – *Hubble Galaxy Classification Scheme* [15]



[15] The University of Alabama, Arts &Sciences – Department of Physics & Astronomy www.astr.ua.edu/keel/galaxies/classify